Exploring Planar Projection of Point Clouds: A Case Study with Cylindrical Objects

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Abstract

Terrestrial laser scanning is widely employed for assessing deformations in structures with diverse geometries, offering a quasi-continuous surface model that accurately reflects current geometric parameters. However, interpreting laser scanning results remains challenging, limiting its adoption as a reliable solution for construction measurements. Point clouds derived from terrestrial laser scanning are often considered supplementary due to difficulties in visualization and precise analysis. In practice and literature, deformation analysis of complex geometric objects typically involves vertical and horizontal cross-sections or color-coded three-dimensional models to illustrate specific deformations. Nevertheless, these methods have inherent limitations, prompting research into projecting point clouds onto a plane for geodetic monitoring. While the mathematical concept of surface projection onto a plane is well-established, applying this to deformation analysis requires a specialized approach, including careful parameter selection and accuracy assessment. Additionally, methods of point cloud filtering, thinning, and classification significantly influence deformation analysis outcomes. This article proposes a solution using cylindrical objects as a case study, demonstrating the method's applicability for monitoring tunnels, vaults, collectors, and other structures.

Keywords: Terrestrial laser scanning, Planar projection, Cylindrical objects

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1 Introduction

Terrestrial laser scanning (TLS) technology provides quasi-continuous information about the surveyed surface in the form of a point cloud with X, Y, Z coordinates, complemented by reflection intensity. It is widely used for monitoring and determining various surface deformations (Harmening et al., 2021; Yang et al., 2021; Gawronek et al., 2019; Kovanič et al., 2019; Liu et al., 2020).

The primary aim of this article is to explore the potential of planar projection of point clouds for identifying deformations in cylindrical surfaces of structures, such as tunnels (Chmelina et al., 2012), water turbine chambers, sewage channels, or building vaults (Dąbrowski and Specht, 2019; Muszyński et al., 2020; Xiang et al., 2021; Ye et al., 2018). The literature contains numerous examples of terrestrial laser scanning applications for curved objects. However, in most cases, the final analysis is limited to generating cross-sections or visualizing deformations on a three-dimensional model. Cross-sections, while useful, allow deformation analysis only in selected profiles rather than across the entire structure. On the other hand, 3D models, despite their advantages, require specialized software for data visualization and often lack readily available tools for measurements along curved surfaces.

For large and complex structures, analysis necessitates selecting specific fragments of the point cloud and conducting localized assessments. Otherwise, the point cloud becomes unreadable due to overlapping points in the current perspective or orthogonal view. Projecting selected surfaces onto a plane is an effective method for presenting measurement results of complex objects using terrestrial laser scanning. This approach is employed both for objects with basic geometries, such as spheres or cylinders (Xu et al., 2019; Pinpin et al., 2021), and for more complex structures, such as telecommunication antennas (Dąbrowski et al., 2019).

Measurement results presented on a plane are easily interpretable and can be analyzed using basic 2D tools. Planar projection of point clouds can be performed using built-in tools in terrestrial laser scanning software or with basic mathematical formulas.

2 Methodology

Expansion of selected surfaces to a plane is a complex task. Each curved surface requires a specific development function and parameter selection. This article focuses on cylindrical surfaces, which appear to be the most commonly used in construction. Figure 1 and formulas (1) and (2) refer to the development of a cylinder, while equation (3) represents the radial deviation from the cylinder form. The value of z_P^R is interpreted as deformations in presented practical applications.

Evaluation of the expansion parameters depends on the spatial structure of the point cloud and has a direct influence on the distortion distribution in the expansion. The optimal choice of the cylinder radius determines the distortion distribution and directly affects the reliability of the final product (Dąbrowski and Specht, 2019).

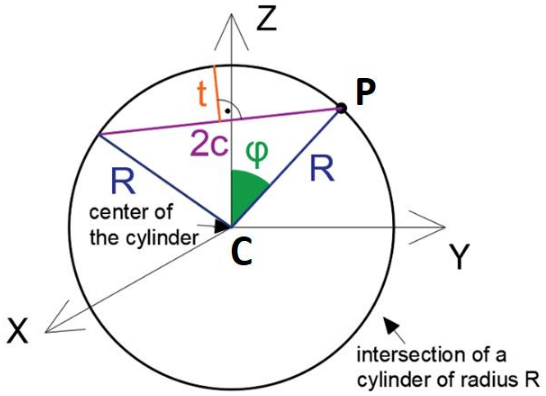

Figure 1. Parameters of the expansion of the cylinder's side surface

$$x_P^R = x_P \tag{1}$$

$$y_P^R = R * \varphi^{rad} \tag{2}$$

$$z_P^R = \sqrt{(y_C - y_P)^2 + (z_C - z_P)^2} - R \tag{3}$$

Where

x_P^R, y_P^R, z_P^R - projected (spatially expanded) point coordinates,
x_P, y_P, z_P - coordinates of the point on the original surface (point cloud),
x_C, y_C, z_C - coordinates of the center of gravity of the original surface (point cloud),
2c – chord,
t - arc arrow,
φ^{rad} - directional angle of point P (angular parameter of the expansion), *where* $\varphi^{rad} = arctg\left(\frac{y_c - y_p}{z_c - z_p}\right)$
R - cylinder radius.

Engineering objects often have surfaces that do not constitute a full cylinder, but only part of it, therefore Figure 1 also shows the chord and arrow of the arc, which can be used to determine the radius based on the formula below.

$$R = \frac{c^2 - t^2}{2t} \tag{4}$$

Formulas for error analysis in the points of the cylinder's projection onto the plane were derived based on Gauss's law for error propagation and are presented in Kowalska (2024). The following assumptions were made:

- the coordinates of the points and the cylinder radius should be treated as direct and independent measurements, allowing the application of Gauss's error propagation law,
- coordinates of the center of gravity of the original surface (point cloud) were treated as flawless,
- the coordinate errors of the points in the expanding cloud are equal:

$\sigma_x = \sigma_y = \sigma_z = \sigma_p$

- the error of the cylinder radius R is equal to σ_R

Accuracy analysis can therefore be based on formulas 5-8.

$$\sigma_{x_p^R} = \sigma_x \tag{5}$$

$$\sigma_{y_p^R}^{\ 2} = \left(\frac{-R\sigma_{y_P}}{\left(1 + \left(\frac{y_c - y_p}{z_c - z_p}\right)^2\right) * (z_c - z_p)}\right)^2 + \left(\frac{R\sigma_{z_P}}{\left(1 + \left(\frac{y_c - y_p}{z_c - z_p}\right)^2\right) * (z_c - z_p)^2}\right)^2 + \left(\sigma_R * arctg\left(\frac{y_c - y_p}{z_c - z_p}\right)\right)^2 \tag{6}$$

$$\sigma_{z_p^R}{}^2 = \left(\frac{-1\,(y_c-y_p)}{\sqrt{(y_c-y_p)^2+(z_c-z_p)^2}}\right)^2 (\sigma_{y_P})^2 + \left(\frac{-1\,(z_c-z_p)}{\sqrt{(y_c-y_p)^2+(z_c-z_p)^2}}\right)^2 (\sigma_{z_P})^2 + (-1)^2(\sigma_R)^2 \quad (7)$$

$$\sigma_{p_p^R}{}^2 = \sigma_{x_p^R}{}^2 + \sigma_{y_p^R}{}^2 + \sigma_{z_p^R}{}^2 \quad (8)$$

Figure 2 illustrates the subsequent steps for determining object changes in shape based on the expansion of the cylinder's side surface onto a plane. The process begins with acquiring raw point clouds representing the examined surface. When measurements are taken from multiple positions, mutual registration of point clouds is required. Cylindrical cross-section objects, such as tunnels, often exhibit a linear and elongated shape, necessitating a specialized approach to registration (Huang et al., 2021).

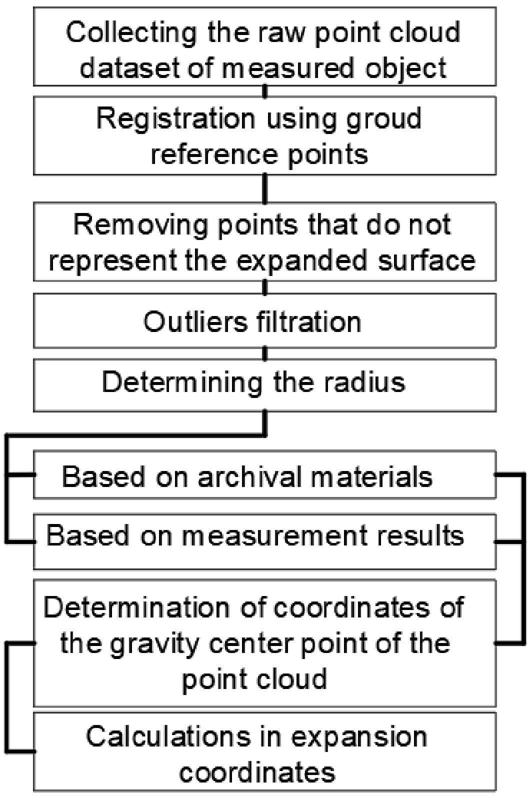

Figure 2. Schematic of the procedure for expanding the point cloud onto a plane

The next step is removing points that do not represent the cylinder's side surface, such as scanned technical infrastructure. Depending on the proportion of unwanted points in the cloud, two approaches can be used. One option is manual removal. The other involves fitting a cylinder to the raw point cloud and then removing significantly deviating points. Although this second method is more automated, it has several statistical limitations, as outlined in the article.

The subsequent step involves eliminating outliers. The processed point cloud can then be expanded onto a plane. This step requires determining the cylinder radius and the axis of gravity as the reference line for the final expansion. The cylinder radius value can be derived from archival materials (e.g., construction plans, as-built surveys) or directly from the point cloud, as demonstrated in Kowalska and Peplinska (2024).

3 Results

For two test objects, a section of the Warsaw metro tunnel (Figure 3) and a section of the communication and inspection gallery of the Rożnów dam (Figure 6), the scanned surface was expanded onto a plane following the procedure outlined in the article. In both cases, the cylinder radius was determined based on the measured point cloud.

3.1. Warsaw Metro tunnel – Line II

The subject of the measurement was a section of the Line II Warsaw metro tunnel (Figure 3). This is a single-track shield tunnel drilled with TBM-type EPB (earth pressure balance) machines, with a diameter of 6.5 m. For the study, measurements were conducted using a Leica RTC360 laser scanner, capable of a scanning speed of up to 2 million points/sec, with a declared scanning resolution of 3 mm, 6 mm, or 12 mm at 10 m, angular accuracy of 18", distance measurement accuracy of 1 mm + 10 ppm, and 3D point position accuracy of 1.9 mm (at 10 m) and 2.9 mm (at 20 m).

The main problem in the experiment of expanding this point cloud onto plane was connected to the cylinder radius value. According to the nominal value, the tunnel radius should be 2.70 m; however, based on the obtained measurements, the average radius was found to be 2.62 m. Using an incorrect radius value caused the horizontally developed surface to stretch and bend.

The results of the expansion and comparison of point clouds using intensity color mapping and corresponding deformation values for the metro tunnel section are presented in Figure 4. The displayed deformations ranged from -0.027 m to 0.033 m. It is important to note that the extreme deformation values occurred at the connections between tunnel segments.

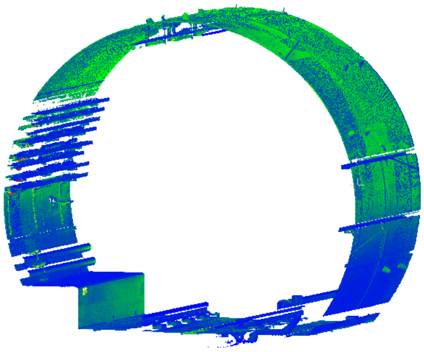

Figure 3. Point cloud depiction of a section of the Warsaw metro tunnel

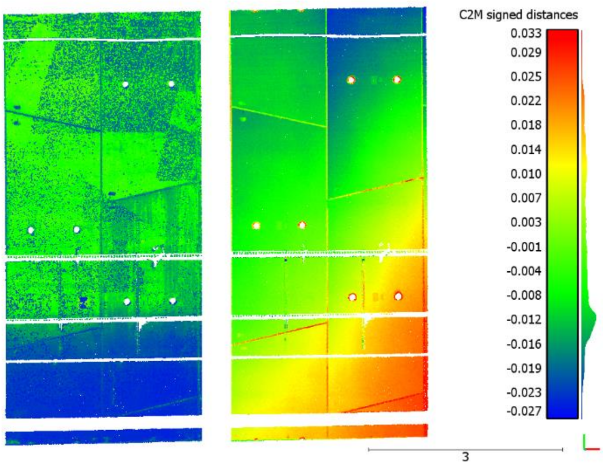

Figure 4. On the right, the expansion of the point cloud in intensity colors, on the right, map of the deformation values for the metro tunnel section

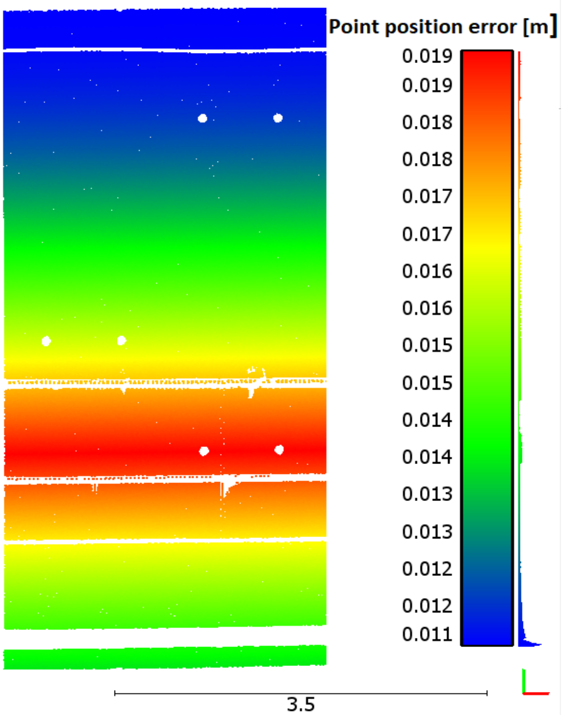

Figure 5. Map of the distribution of average errors in point positions on the expansion in three-dimensional space calculated using equation 8

An accuracy analysis of the point position in 3D space was performed based on formulas (5-9). Figure 5 represent fragment of map of the distribution of average errors in point positions on the expansion in three-dimensional space

Table 1. Mean, maximum and minimum values of average errors in point positions on the expansion in three-dimensional space calculated using equations 5-8

	$\sigma_{x_p^R}$	$\sigma_{y_p^R}$	$\sigma_{z_p^R}$	$\sigma_{P_p^R}$
Mean	0.010	0.014	0.014	0.022
Max	0.010	0.016	0.014	0.024
Min	0.010	0.011	0.014	0.011

It was assumed that, in accordance with the Leica RTC360 instrument parameters, coordinates were determined with an accuracy of 0.003 m, and the cylinder radius with an accuracy of 0.01 m. Based on Table 1 and Figure 5 the average errors in point positions in three-dimensional space ranged between 1.1 cm and 2.4 cm, which is a satisfactory result, demonstrating that with this level of accuracy, deformations exceeding 2 cm can be reliably detected.

3.2. Communication and inspection gallery of the Rożnów Dam

The communication and inspection gallery of the Rożnów Dam features a cylindrical cross-section. It was scanned using the MS60 scanning total station. This instrument scans at a speed of 30,000 Hz (30,000 scans per second) and operates in four modes, characterized by the following maximum ranges and outliers levels: 60 m / 3 mm; 150 m / 1.5mm; 300 m / 1.0 mm and 1000 m / 0.6 mm. The study utilized a fragment of the point cloud from a single station (Figure 6).

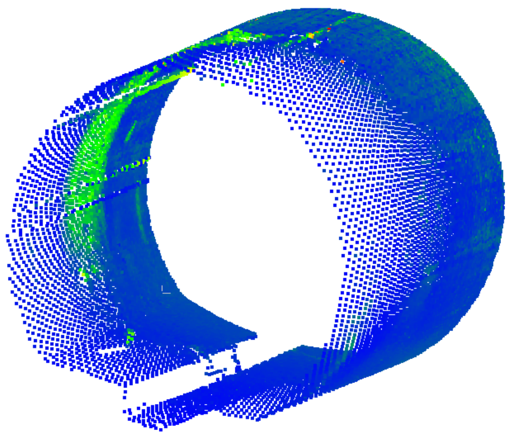

Figure 6. Point cloud representing a section of the Rożnów Dam gallery

Figure 7 presents the results of the point cloud expansion using intensity color mapping on the left and deformation value mapping on the right for the communication and inspection gallery of the Rożnów Dam.

The conducted comparisons highlighted the significance of determining deformations based on the expanded point cloud. The expanded point cloud is compared to a plane. Most point cloud processing software fits a plane using the least squares method. This approach can obscure small deformations. Another issue with using the least squares method is the presence of unfiltered outliers. For example, in Figure 7 (right), isolated blue squares likely represent outliers not removed during the preparation of the point cloud for expansion. These points affected the surface fitting process using the least squares method. Figure 8 shows a histogram of the point-to-plane distance distribution, fitted using the least squares method.

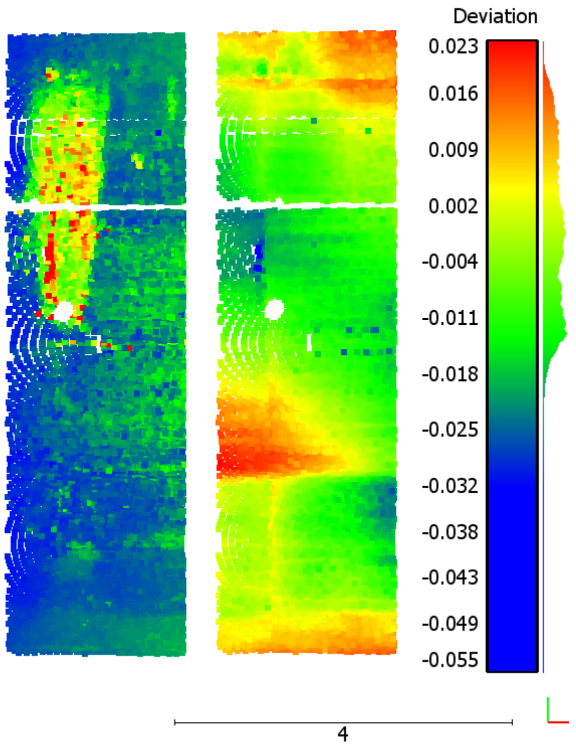

Figure 7. Left: results of point cloud expansion with intensity colors; Right: results with deformation values for the communication and inspection gallery of the Rożnów Dam

The standard deviation for the analyzed data was determined as σ = 0.01. An analysis was then conducted to assess the filtering potential of the point cloud based on the calculated distances from a plane fitted using the least squares method. The adopted significance level of 2.5 established cut-off thresholds at 2.5σ, enabling the removal of outlier points that were most likely measurement noise, as indicated by the histogram. As a result of the experiment, approximately 0.1% of the points identified as outliers were removed from the point cloud, and a new histogram of point-to-plane distances was generated using the least squares method (Figure 9). The histogram in Figure 9 indicated that measurement outliers were still present in the point cloud. Therefore, for further analysis, a new maximum deformation threshold of 0.020 m (significance level of 2.0) was adopted, leading to the classification of 2.6% of the points as outliers. The results of these two processing variants are shown in Figure 10.

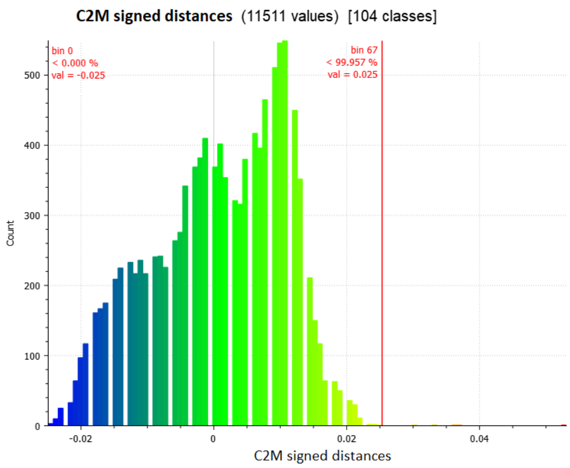

Figure 8. Histogram of point-to-plane distance distribution using the least squares method – Variant 1 (communication and inspection gallery of the Rożnów Dam)

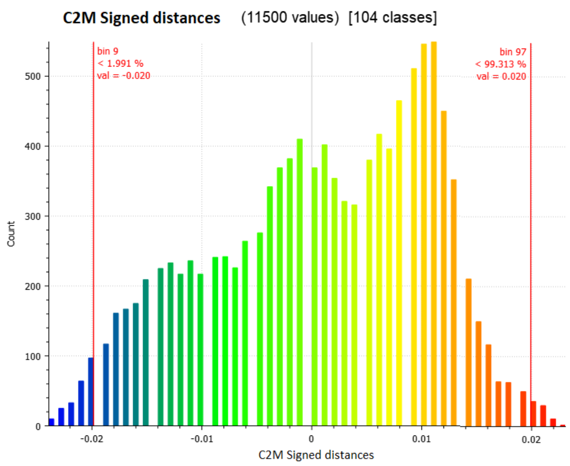

Figure 9. Histogram of point-to-plane distance distribution using the least squares method – Variant 2 (communication and inspection gallery of the Rożnów Dam)

Figure 10 compares the planar expansion results for the point cloud fragment of the Rożnów Dam communication and inspection gallery under three conditions: without removing points, with points removed based on the threshold from Figure 8, and with points marked for removal (black color) based on the threshold from Figure 9. The analysis concluded that the 0.025 m threshold was insufficient, as individual red points remained visible in the processed data. Conversely, adopting the 0.020 m threshold led to the removal of too many points that accurately represented the studied surface. It should be emphasized that the deviations determined from the plane actually represent deviations from the cylindrical surface in 3D. The analyzed point clouds correspond to smooth cylindrical surfaces, with fragments representing structural elements excluded from the analysis. Adopting a significance level of 2.5 resulted in the removal of individual outlier points, whereas at level 2, entire fragments of the point cloud were identified, potentially indicating deformations in those areas. This analysis underscores the challenges of selecting appropriate parameters for removing inaccurately measured points.

Terrestrial laser scanning provides quasi-continuous information about a studied surface in the form of high-density point clouds. However, measurement outliers is inherent in these data, potentially leading to erroneous analysis results. Therefore, it is crucial to carefully analyze and consider various aspects of applied solutions.

When determining deformations, selecting the appropriate points that accurately represent the analyzed surface is just as important as choosing the reference surface. By expanding a point cloud onto a plane, it is possible to analyze the distance of points either from an horizontal plane or from a plane fitted using the least squares method. Figure 11 presents a histogram of differences between deformation values determined with reference to a horizontal plane and a plane fitted using the least squares method. The discrepancy between the results of the two approaches ranged from -0.015 m to 0.015 m, indicating relatively minor differences in this case. However, it is essential to note that for surfaces with greater deformations, these discrepancies would be larger. The authors suggests that expanded surfaces should be compared to ideally horizontal planes. This approach avoids averaging distance values and minimizes the global impact of undetected outliers on the analysis results. Figure 12 illustrates a map of differences in determined deformations based on a horizontal plane and a plane fitted using the least squares method. The results presented on the deformation map (Figure 12) exhibit a pattern of diagonal stripes, which may suggest the influence of measurement outliers, as seen in Figure 10, on the plane fitting using the NMK method. A significant concentration of outliers in the upper left corner of the analyzed surface caused an oblique inclination of the plane fitted using the NMK method.

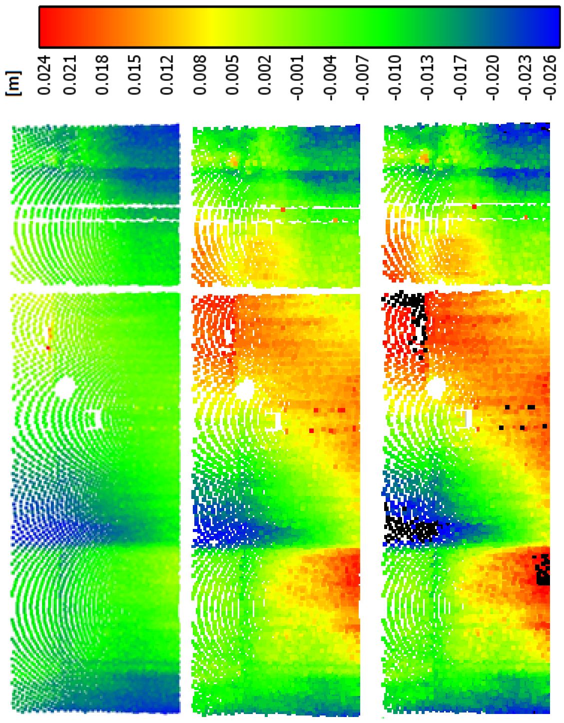

Figure 10. Comparison of deformation maps of point clouds filtered based on the distance of points from the surface fitted using the least squares method, black color - points marked for removal

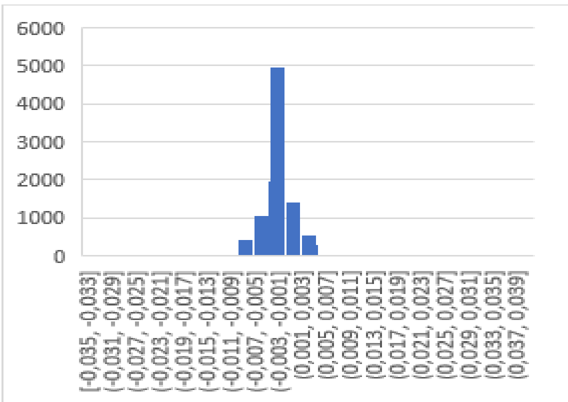

Figure 11. Histogram of differences in determined deformations based on a horizontal plane and a plane fitted using the least squares method

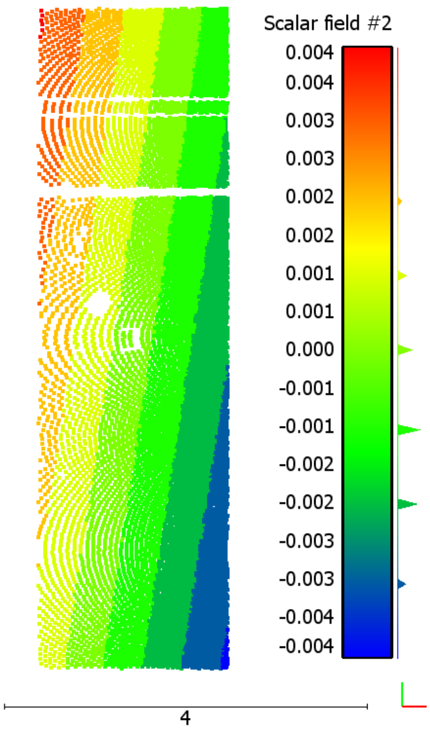

Figure 12. A map of the differences between deformations based on a horizontal plane and a least squares fitted plane

4 Conclusions

Terrestrial laser scanning is currently one of the most widely used measurement techniques, and point clouds are utilized in numerous geodetic and related tasks. Despite their extensive data, proper processing is crucial to ensure accurate and reliable results, a point particularly emphasized by the authors. Expanding cylindrical surfaces onto a plane enables deformation analysis in the form of projected coordinates. Furthermore, it enhances the readability of analyzed point clouds for symmetrical objects, where overlapping points occur in a three-dimensional view. Additionally, planar expansion facilitates the easy generation of two-dimensional drawings, which are essential for some engineering geodesy tasks.

This article discusses methods for expanding cylindrical surfaces onto a plane, with a detailed analysis of selecting a reference surface for determining deformations. The results for a plane fitted to the point cloud using the least squares method (the most common approach in software dedicated to point cloud analysis) were compared with those for an ideally horizontal plane. In the analyzed case, the observed discrepancies were negligible; however, in Author's opinion that these discrepancies will increase for more deformed objects. However, this requires verification and further research to confirm these assumptions. It is also important to remember that the expansion of a surface onto a plane has internal accuracy, which contributes to the calculated deformations. Regardless of whether the expansion method is automatic or manual, the accuracy of determining the cylinder radius and the object's axis will impact the quality of the analysis.

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