Detection analysis of displaced connection points for a different type of engineering survey networks connections

Sławomir ŁAPIŃSKI[1,*], and **Piotr MĄKOWSKI**[1]

[1] *Warsaw University of Technology, Faculty of Geodesy and Cartography, Warsaw, Poland, (slawomir.lapinski@pw.edu.pl; piotr.makowski3.stud@pw.edu.pl)*

[*] *corresponding author*

Abstract

The paper aims to perform analyses based on the detectability of displaced connection points depending on the engineering survey networks' chosen type of connection to these points. An engineering survey network is closely related to an engineering object. It is designed not only for staking out but also for subsequent inventory measurements and object control. In the research, the application of the three most popular types of network connection, i.e. rigid - not taking into account the accuracy of the reference points, stochastic and non-distorting - taking into account the accuracy of the reference points were used. The calculations were performed using the mathematical model of the observation adjustment using the least squares method. The diagnostic tests (global test and local test) will be used to detect "gross errors" in observations directly related to the displaced connection point. Within the research, an observation system for the higher level network (external network) and engineering survey network were created with different connection types, changing the number of connection points and the number of observations to those points. It was made to analyse different variants of observation configurations. Additionally, based on coordinates adjusted before and after displacing the given connection point, a detailed analysis of geolocation changes in the engineering survey network was made. It is possible to detect displaced points using a different type of connection engineering survey networks. However, an adequate number of connection points and observations of these points are needed.

Keywords: network connection, engineering surveying network, diagnostic tests

1 Introduction

Construction of a new facility or installation requires developing an implementation plan to define the project's location precisely. However, establishing a geodetic control network is essential for determining the layout of the new investment in the field. This network serves as the basis for setting out all project elements, ensuring they align with the implementation plan. The geodetic control network plays a critical role in the investment process, as it determines the accuracy of the positioned elements. Designing this network involves analysing the implementation plan to ensure that the network's points remain stable throughout the entire investment process. Designing geodetic networks involves optimising configurations to achieve precise, reliable, and cost-effective results. This process includes different stages such as Zero-Order Design (ZOD) for selecting an optimal coordinate system, First-Order Design (FOD) for determining point positions and observation plans, Second-Order Design (SOD) for assigning measurement weights, and Third-Order Design (THOD) for improving existing networks (Grafarend, 1974; Grafarend and Sansò, 2012; Kuang, 1991; Odziemczyk, 2024). Advanced methods like Genetic Algorithms (GA) and Particle Swarm Optimisation (PSO) are used to enhance design efficiency by simulating natural processes such as evolution or group behaviour, providing better solutions than traditional trial-and-error approaches (M. Doma and Al Shouny, 2011; M. I. Doma, 2013; Mrówczyńska and Sztubecki, 2019, 2021; Singh et al., 2016). Mathematical models using nonlinear programming with constraints on precision and reliability ensure the network meets specific requirements, like fault monitoring. Examples of such optimisation include configuring fault-line observation points for maximum accuracy while minimising cost.

This article analyses the detectability of displaced reference points based on the method chosen for connecting the engineering network to these points. Three connection types are applied: rank-based connection, which does not account for accuracy in reference points; stochastic and non-distortion connection, which incorporates accuracy of reference points.

2 Mathematical model for connection of engineering survey network

This chapter discusses the mathematical model for the adjustment of observations using the least squares method and the definition of the connection of engineering survey networks. It also includes information on the diagnostic tests conducted and the global and local tests used to detect outlier observations associated with reference points. It is presented here in a standardised form of the mathematical model, as outlined by Prószyński (2010)

$$\mathbf{A}_s\mathbf{x} = \mathbf{y}_s + \mathbf{v}_s \text{ ; } \mathbf{C}_s \tag{1a}$$

$$\mathbf{S}\mathbf{x} = \mathbf{b} \text{ ; } \mathbf{C}_\mathbf{b} \tag{1b}$$

where
$\mathbf{x}(u\times 1)$, $\mathbf{y}_s(n\times 1)$, $\mathbf{A}_s$ $(n\times u)$, rank $\mathbf{A}_s$ = u-d (d - network defect), $\mathbf{C}_s$ $(n\times n)$ (pos. definite), $\mathbf{S}$ $(w\times u)$, $w\geq d$, (w-number of condition) rank $\mathbf{S}$ = w, rank $[\mathbf{A}_s^T \quad \mathbf{S}^T]$ = u, $\mathbf{b}$ - the vector of increments in network location features with covariance matrix $\mathbf{C}_\mathbf{b}$, $\mathbf{b}(w\times 1)$.
The matrix is presented in a partitioned form, displaying the sub-vectors x_1 and x_2 , which correspond to the connection points and the remaining network points, respectively (Prószyński and Łapiński, 2018).

$$\mathbf{A}_{1,s}\mathbf{x}_1 + \mathbf{A}_{2,s}\mathbf{x}_2 = \mathbf{y}_s + \mathbf{v}_s \text{ ; } \mathbf{C}_s \tag{2a}$$

$$\mathbf{S}_1\mathbf{C}_{\mathbf{X},1}^{-1}\mathbf{x}_1 + \mathbf{0}\cdot\mathbf{x}_2 = \mathbf{b}\ (\mathbf{b} = \mathbf{0};\ \mathbf{C}_\mathbf{b}) \tag{2b}$$

where $\mathbf{C}_{\mathbf{X},1}$ $(u_1\times u_1)$ (pos. definite) covariance matrix for connection points. Eq. (2b) describes the conditions that must be imposed on the connection points. This partitioning is crucial for differentiating between the parameters directly related to the reference points and those associated with the rest of the network.

1. Rank-based connection of engineering survey networks

In rank-based connection, connection points are assumed to be errorless. This assumption implies the invariability of the positions of the points to which the network is referenced. Consequently, these reference points are assigned coordinate corrections equal to zero during adjustment. The adjustment can be performed in two ways: (1) standardised adjustment model, (2) reduction of Eq. (2a) using the matrices $\mathbf{A}_{1,s}$ and $\mathbf{x}_1$ (the removal of the information related to the connected points). In this case, Eq. (2b) will take the form.

$$(1)\ [\mathbf{A}_{1,s}\ \mathbf{A}_{2,s}]\cdot\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} = \mathbf{y}_s + \mathbf{v}_s \text{ ; } \quad \mathbf{C}_s \text{ ; } \mathbf{C}_{\mathrm{X},1}=\mathbf{0} \tag{3a}$$

$$(2)\ [\mathbf{A}_{2,s}]\cdot[\mathbf{x}_2] = \mathbf{y}_s + \mathbf{v}_s \text{ ; } \mathbf{C}_s \tag{3b}$$

minimising the form

$$\Omega = \mathbf{v}_s^T\mathbf{C}_s^{-1}\mathbf{v}_s \tag{4}$$

2. Stochastic connection of engineering survey networks

This method incorporates the uncertainties of connection points into the network's adjustment by assigning weights to reference points based on their known or estimated accuracy.

$$(1)\ [\mathbf{A}_{1,s}\ \mathbf{A}_{2,s}]\cdot\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} = \mathbf{y}_s + \mathbf{v}_s \text{ ; } \quad \mathbf{C}_s \text{ ; } \mathbf{C}_{\mathrm{X},1}\neq\mathbf{0} \tag{5}$$

minimising the form

$$\Phi = \mathbf{v}_s^T\mathbf{C}_s^{-1}\mathbf{v}_s + \mathbf{x}_1^T\mathbf{C}_{\mathbf{X},1}^{-1}\mathbf{x}_1 \tag{6}$$

being sometimes termed as "Tienstra connection"(Tienstra, 1956).

3. Non-distorting connection of engineering survey networks

The non-distorting connection (NDC) represents a specialised network connection designed to meet the rigorous demands of high-accuracy and high-reliability engineering surveys. Conceptually, it is implemented as a two-step process. The first step involves the adjustment of the newly constructed network within its own reference system, ensuring internal consistency and precision. The second step comprises a Helmert transformation of the adjusted coordinates into the specified connection points, with the crucial condition of preserving the scale of the new network. By maintaining the network's original scale during its integration with the connection points, this approach minimises the potential influence of errors in the connection points' coordinates on the geometry of the new network. The study (Prószyński, 1986) presents a joint adjustment model for NDC together with the relevant accuracy analysis, both in the network's own reference system and the external reference system. The observation model is the same as Eq. (2a). Based on Eq. (2b), the condition for reference points

$$\begin{gathered}\mathbf{S}_1\mathbf{C}_{X,1}^{-1}\mathbf{x}_1 + \mathbf{0}\cdot\mathbf{x}_2 = \mathbf{b} \\ (\mathbf{b} = \mathbf{0};\ \mathbf{C}_b \neq \mathbf{0};\ \mathbf{C}_{X,1} \neq \mathbf{0})\end{gathered} \tag{7}$$

minimising the two independent form

$$\begin{gathered}\Omega = \mathbf{v}_s^T\mathbf{C}_s^{-1}\mathbf{v}_s \\ \Psi = \mathbf{x}_1^T\mathbf{C}_{X,1}^{-1}\mathbf{x}_1\end{gathered} \tag{8}$$

4. Statistical tests

Statistical tests are used to assess the consistency of the results obtained: the global test Eq. (9) and the local test Eq. (10). The first of these tests is the null hypothesis of the coefficient of variance. If this value equals one, the assumed a priori errors were correct, there are no coarse errors in the observational material, and the assumed network model is correct. If the value is more than one, there are outliers in the observational material, or the network model is incorrect. A local test (unified correction test) makes it possible to identify an individual outlier observation.

$$\begin{gathered}\sigma_0 \leq \sigma_{0,\,critical} \\ \sigma_{0,\,critical} = \sqrt{\frac{\chi^2_{f,\alpha}}{f}}\end{gathered} \tag{9}$$

$$\begin{gathered}|u_i| \leq u_{i,\,critical} \\ u_i = \frac{V_i}{\sigma_{V,i}};\ \sigma_{\hat{v},i} = \sqrt{\{C_V\}_{i,i}}\end{gathered} \tag{10}$$

3 Empirical tests

The first stage of the experiment involved the creation of an engineering survey network, which was adjusted using "free" type conditions along with observations to reference points, and a higher-order reference network was adjusted with a point-line condition. The network layout is shown in Figure 1.

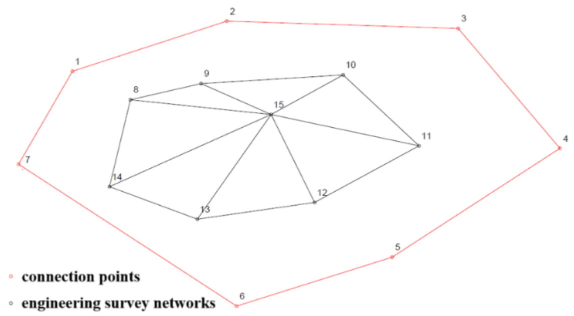

Figure 1. Engineering survey network and higher-order reference network

Subsequently, observational systems were developed with different configurations of observations to the reference points, along with the determination of measurement accuracy and the creation of a "free" term matrix derived from a normal distribution. After completing the aforementioned stages, the network was referenced using rank-based, stochastic, and NDC methods. At this stage, results were obtained, which served as the benchmark for further analysis. However, the study aimed to verify whether the referencing methods could detect the displacement of a higher-order reference point to which the realisation network would be referenced. Such a displacement would not be detected during the measurement of the realisation network, as it was adjusted using "free" conditions, where corrections to the coordinates are based on the observations. Therefore, the global and local tests would not detect gross errors. As a result, the graphs for the local tests and σ_0 will be identical.

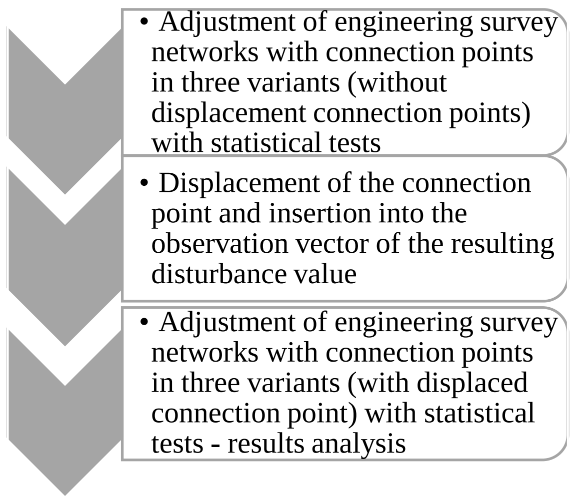

Figure 2. The analysis strategy

The next step involved introducing disturbances into the "free" term matrix based on the reference points previously established known displacement values. With this matrix, the network was readjusted, followed by its referencing, and compared to the benchmark values. By comparing the results, one could determine whether the referencing method detected the point displaced by a known value. The analysis strategy is presented in Figure 2.

3.1 Adjustment of the higher-order geodetic network

The adjustment was made to this network using point-to-line condition. Points numbered 5 and 6 were chosen for this. The network is in the form of a closed polygon by which the accuracy of the position of the points in the network is higher. This adjustment will remain constant throughout the experiment process. The observation is shown in Table 1.

Table 1. Observations of the higher-level geodetic network

No.	centre	left	right	$\sigma_{apriori}$ [m,g]
1	1	2	0	0.005
2	2	3	0	0.005
3	3	4	0	0.005
4	4	5	0	0.005
5	5	6	0	0.005
6	6	7	0	0.005
7	1	2	7	0.0015
8	2	1	3	0.0015
9	3	4	2	0.0015
10	4	5	3	0.0015
11	5	4	6	0.0015
12	6	7	5	0.0015
13	7	6	1	0.0015

Figure 3 shows the accuracy characteristics of the positions of the points after adjustment in the form of confidence ellipses.

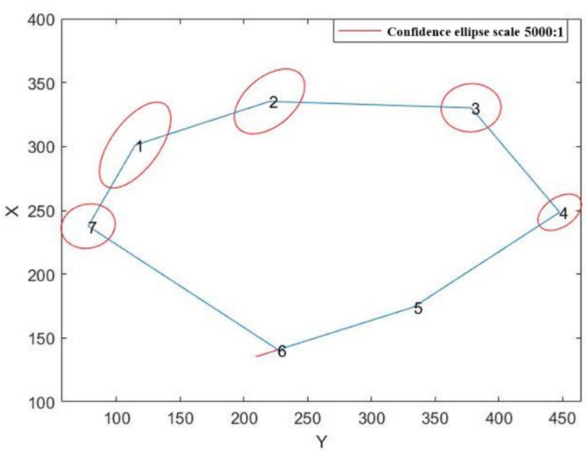

Figure 3. Higher-order reference network with confidence ellipses

3.2 Connection the engineering survey network to two reference points

In this configuration, the engineering survey network was connected to points 3 and 4, and then a displacement of $\Delta x = 0.012$m and $\Delta y = 0.002$m was given to point number 3. The network consists (Figure 4) of 18 distance observations (5 mm accuracy) and 18 angle observations (1.5 mgon accuracy).

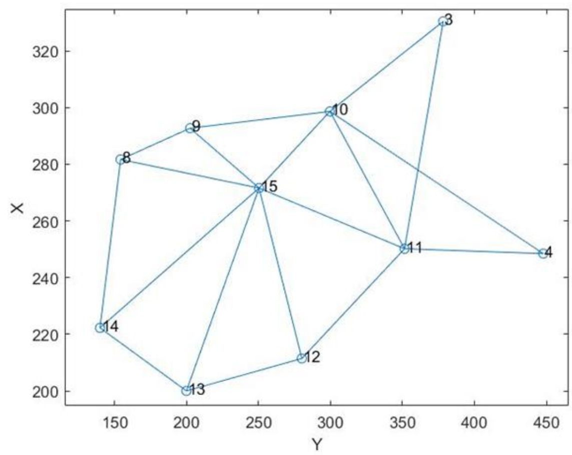

Figure 4. The network with two connection points - configuration

Figures 5-7 present the numeral analyses carried out for the two connection points. Figure 7 shows results only in the second step in NDC.

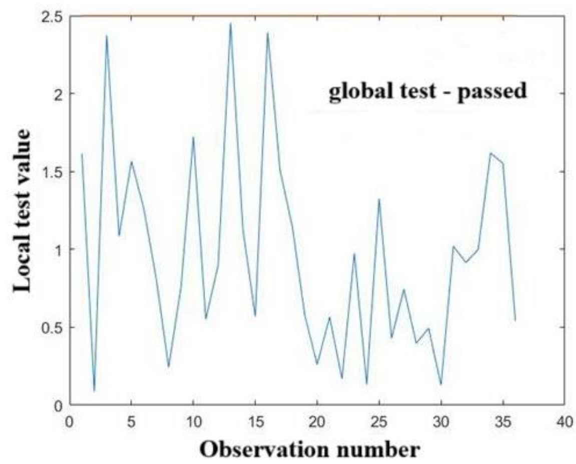

Figure 5. Value of statistical tests for rank-based connection after displaced connection point

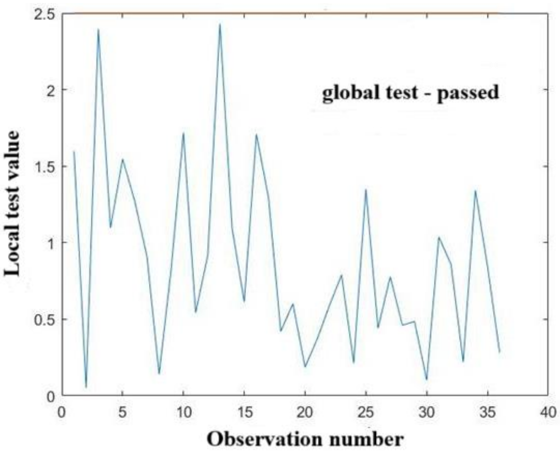

Figure 6. Value of statistical tests for stochastic connection after displaced connection point

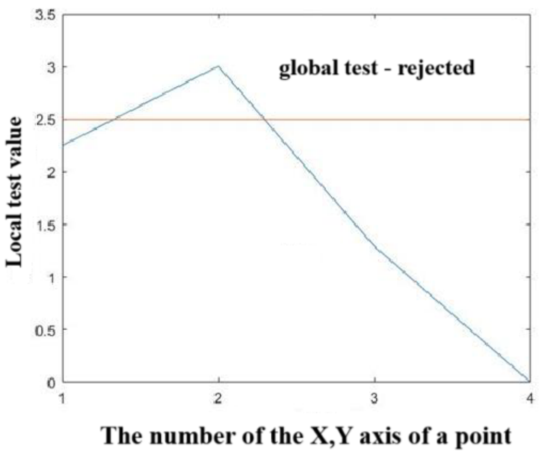

Figure 7. Value of statistical tests for NDC (second step) after displaced connection point

The analysis results of adjustment rank-based connection for the engineering survey network, as well as local and global tests for adjustment, fail to detect gross errors due to the displaced connection point. This limits the ability to identify point displacements without comparison to prior epochs. Similarly, stochastic reference adjustment did not identify displacements in the tests but exhibited significant shifts in realisation points. Only non-distorting reference adjustment detected reference point displacements; however, only after the Y-axis component (the main displacement value was after the X-axis component).

3.3 Connection the engineering survey network to three reference points

More observations of the reference points (2,3,4) were included for this configuration. The total number of observations is 42 (21 linear observations (5 mm accuracy) and 21 angular observations (1.5 mgon accuracy)), according to Figure 8. Displacement was included at point 3 with values $\Delta x = 0.018$m and $\Delta y = 0.013$m.

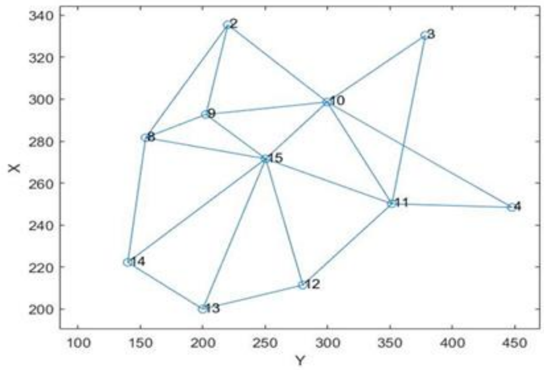

Figure 8. The network with three connection points - configuration

Figures 9-11 show the numeral analyses carried out for the three connection points according to the scheme shown in Figure 2.

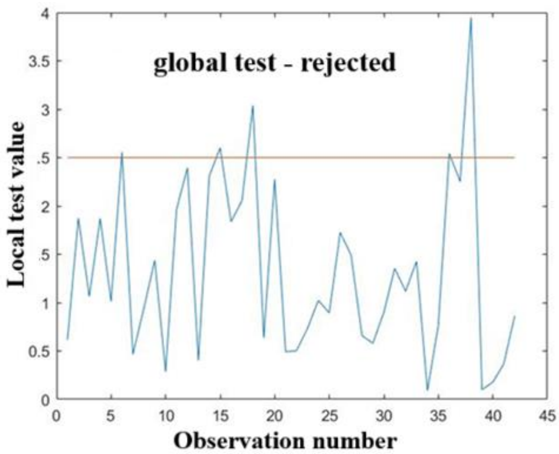

Figure 9. Value of statistical tests for rank-based connection after displaced connection point

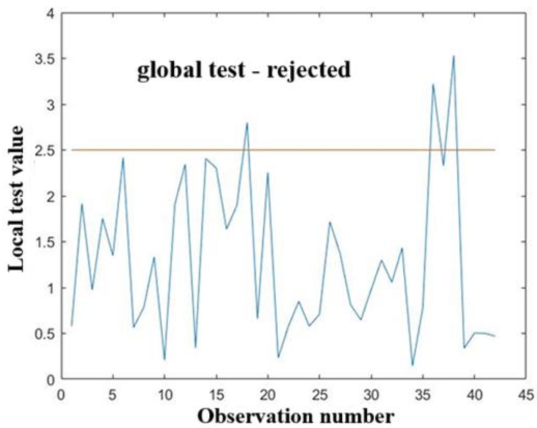

Figure 10. Value of statistical tests for stochastic connection after displaced connection point

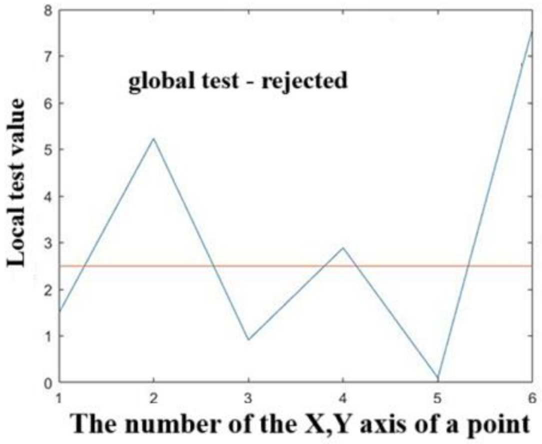

Figure 11. Value of statistical tests for NDC (second step) after displaced connection point

The results of the numerical analyses confirm the possibility of detecting and localising the displaced point based on the global and local observation tests, respectively.

3.4 Connection the engineering survey network to four reference points

Four network points were connection points in the third variant of the geodetic network (Figure 12). In this configuration, displacements were given to two points: 1($\Delta x = 0.012$m and $\Delta y = 0.002$m) and 4 ($\Delta x = 0.011$m and $\Delta y = 0.000$m). The network consists of 22 distance observations (5 mm accuracy) and 22 angle observations (1.5 mgon accuracy).

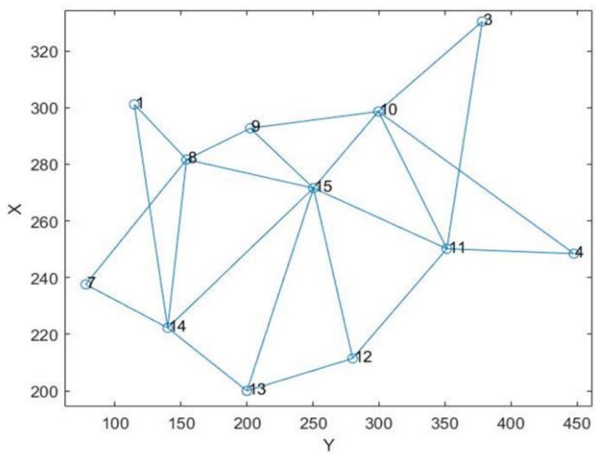

Figure 12. The network with four connection points - configuration

Figures 13-15 show the numeral analyses for the four connection points after displaced points 1 and 4.

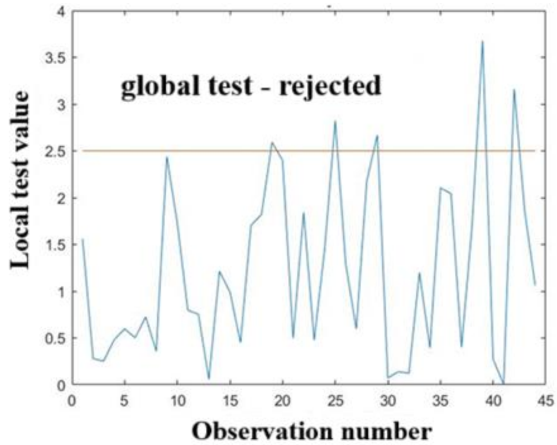

Figure 13. Value of statistical tests for rank-based connection after displaced connection points

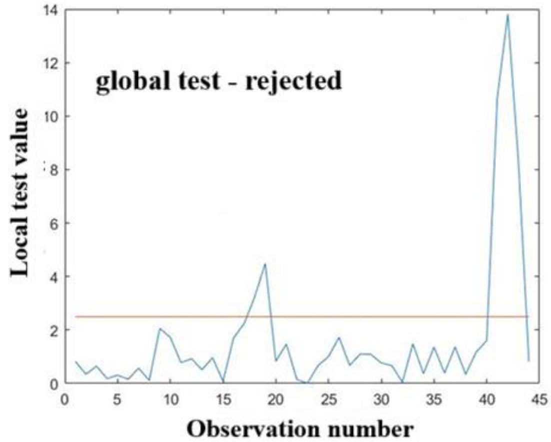

Figure 14. Value of statistical tests for stochastic connection after displaced connection points

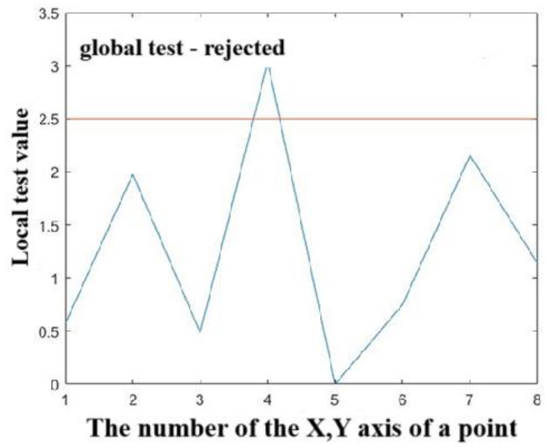

Figure 15. Value of statistical tests for NDC (second step) after displaced connection points

In the rank-based connection, the global and local tests detected gross errors in the observations of the displaced reference points. In the stochastic connection, gross errors in the observations are also apparent. However, when two points are displaced, it is not possible to correctly identify the displaced points due to the uniform distribution of the points resulting from the characteristics of the reference. The NDC is the only connection where the displacement values of the reference points are visible. In this reference, coarse errors were also detected in the diagnostic tests.

4 Conclusion

The analyses confirmed that the three most popular connections used in surveying are capable of detecting displacements on these points despite not detecting them when adjustment observations of the realisation network. This is evident in global and local tests, making it possible to detect displaced points based on disturbances in the observations. An important element in displacement detection is good network controllability, i.e., an adequate number of redundancy observations. The adjustment of the higher-order network so that the reference points are not error-free from the point-to-line condition because the disturbance may not be detected in such a case. The number of connection points also greatly impacts the detection of their movement since the major geolocation changes caused by the disturbance are obtained in the reference of two points. The most advantageous referencing method regarding time and cost is referencing three reference points. This is because if we have a sufficient number of excess observations, we can delete incorrect observations and then make a reference to the two remaining points. A larger number of points will result in more work with similar detection. However, the values of this displacement will be known in a rather very approximate form. This will also improve the control of the non-distorting reference. On the other hand, a smaller number will result in a single-point reference when one point is displaced, making it very unfavourable. It can also be said that the in-row reference is not very "sensitive" since the global test detects gross errors. Still, it is not always possible to identify a disturbed observation. Geodetic network connection allows us to detect the displacement of reference points. This is very important because failure to detect a given disturbance earlier would distort our entire network and, as in the case of a non-distorting reference, a geolocation change.

References

Doma, M., and Al Shouny, A. (2011). A new method for designing the optimum geodetic networks using genetic algorithms. *Journal of Engineering and Applied Science*, *58*, 109–125.

Doma, M. I. (2013). Particle Swarm Optimisation in Comparison with Classical Optimisation for GPS Network Design. *Journal of Geodetic Science*, *3*(4). https://doi.org/10.2478/jogs-2013-0030

Grafarend, E. W. (1974). Optimisation of Geodetic Networks. *The Canadian Surveyor*, *28*(5), 716–723. https://doi.org/10.1139/tcs-1974-0120

Grafarend, E. W., and Sansò, F. (2012). *Optimisation and design of geodetic networks*. Springer Science \& Business Media.

Kuang, S. (1991). *Optimisation and Design of Deformation Monitoring Schemes. Ph.D. dissertation* (Issue 157). University of New Brunswick, Department of Surveying Engineering.

Mrówczyńska, M., and Sztubecki, J. (2019). The use of evolutionary algorithms for designing an optimum structure of a geodesic measurement and control network. *MATEC Web of Conferences*, *262*, 07008. https://doi.org/10.1051/matecconf/201926207008

Mrówczyńska, M., and Sztubecki, J. (2021). The network structure evolutionary optimisation to geodetic monitoring in the aspect of information entropy. *Measurement*, *179*, 109369. https://doi.org/10.1016/j.measurement.2021.109369

Odziemczyk, W. (2024). Comparison of selected reliability optimisation methods in application to the second order design of geodetic network. *Journal of Applied Geodesy*, *18*(2), 223–236. https://doi.org/10.1515/jag-2023-0024

Prószyński, W. (1986). Accuracy analysis for non-distorting connection of engineering survey network. *XVIII International Congres FIG*.

Prószyński, W. (2010). Another approach to reliability measures for systems with correlated observations. *Journal of Geodesy*, *84*(9), 547–556. https://doi.org/10.1007/s00190-010-0394-2

Prószyński, W., and Łapiński, S. (2018). Reliability analysis for non-distorting connection of engineering survey networks. *Survey Review*, 1–6. https://doi.org/10.1080/00396265.2018.1425605

Singh, V., Dwivedi, R., Dikshit, O., and Singh, A. K. (2016). First-Order Design of GPS Networks Using Particle Swarm Optimisation. *Journal of Surveying Engineering*, *142*(3). https://doi.org/10.1061/(ASCE)SU.1943-5428.0000176

Tienstra, J. M. (1956). *Theory of the adjustment of normally distributed observations*. Amsterdam : Argus. http://lib.ugent.be/catalog/rug01:000474849