Appropriate strategy for GB-RAR measurements - One radar is not sufficient

Milan TALICH[1,*], **Jan HAVRLANT**[1], **Lubomír SOUKUP**[1], **Tomáš PLACHÝ**[2], **Michal POLÁK**[2], **Pavel RYJÁČEK**[3], and **Vojtěch STANČÍK**[3]

[1] *The Czech Academy of Sciences, Institute of Information Theory and Automation, Prague, Czech Republic, (Milan.Talich@utia.cas.cz, havrlant@utia.cas.cz, soukup@utia.cas.cz)*

[2] *Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague, Czech Republic, (plachy@fsv.cvut.cz, polak@fsv.cvut.cz)*

[3] *Department of Steel and Timber Structures, Faculty of Civil Engineering, Czech Technical University in Prague, Czech Republic, (pavel.ryjacek@fsv.cvut.cz, vojtech.stancik@fsv.cvut.cz)*

[*] *corresponding author*

Abstract

Over the past 10 years, ground-based radar interferometry has become a frequently used technology for determining dynamic deflections of bridge structures induced by vehicle passages. When measuring with only one radar device, the so-called Interpretation Error (EI) considerably rises. When using two radars, it is possible to simultaneously determine, for example, vertical and longitudinal displacements and to eliminate the Interpretation Error. The aim of the article is to inform about a suitable strategy for determining dynamic and quasi-static response of bridge structures based on the accuracy analysis of measurement by two radars. The necessary theory for displacements determination by means of two radar devices is presented. This is followed by an analysis of errors when measuring with only one radar. The accuracy of the resulting displacements by simultaneous measurement with two radars is also mentioned. The practical example of bridge structure displacements determination by measuring with two radar devices in the field is presented. The key contribution of the paper is the possibility to estimate and plan in advance the achievable accuracy of the resulting displacements for the given radar configurations in relation to the bridge structure.

Keywords: Interferometric radar; GB-RAR, Remote measurements, Bridge monitoring, Dynamic vertical and horizontal displacements, Measurement accuracy analysis

1 Introduction

Over the past 10 years, ground-based radar interferometry with real aperture radar (GB-RAR or GB-InRAR) has become a frequently used technology for determining dynamic deflections of bridge structures induced by vehicle passages. The radar interferometry method allows to measure real-time deflections for short- and long-term loads in normal traffic (e.g., the passage of vehicles or vice versa standing columns of vehicles or static load tests). Furthermore, it can dynamically capture and detect frequency and amplitude of vibration of the monitored object in the frequency range from approximately 0.0 to 80 Hz. This method provides to determine the deflection size with precision better than 0.1mm. Deflections of a bridge can be simultaneously determined at multiple locations. It is possible to obtain both general and detailed information on the behavior of the structure under its dynamic load. For example, on the bridge of the length of 100 m there is possible to simultaneously monitor up to about 100 points. The basic principles and examples of the use of GB-RAR technology for determining deflection of bridges are given, for example, in Pieraccini at al. (2006), Gentile and Bernardini (2010) and Liu at al. (2015). An example of the use of GB-RAR technology to determine the deflections of metal rail bridge constructions caused by both temperature changes and vehicle passages (dynamic loads) is presented in Talich (2018b).

However, this technology is very often also used for monitoring of further objects. For example, the monitoring of communications towers and urban buildings are described in Luzi at al. (2017) or Talich (2018a) and monitoring of water tower reservoirs, factory chimneys, and wind power plant pylons is given in Talich (2017). The joint use of a terrestrial laser scanner (TLS), configured in line scanner mode, and a GB-RAR technology for monitoring of vibration frequencies and oscillation amplitudes of tall structures is presented in Artese and Nico (2020). The comparisons of the GB-RAR technology and technology using accelerometers for dynamic monitoring of large structures and for monitoring of bridges are given in Pieraccini at al. (2008) or Akbar (2021). A review in the field of GNSS technology use for dynamic structural health monitoring together with other technologies such as accelerometers and RTS (robotic total stations) is presented in Yu at al. (2020).

This contribution is focused on the measurement of deflections of bridges by two IBIS-FS interferometric radars of the Italian manufacturer IDS - Ingegneria Dei Sistemi. More details about this instrument are, e.g., in Gentile and Bernardini (2010).

One of the basic shortcomings of the GB-RAR method is that the radar measures only line of site (LOS) displacements in the direction of intent and these are recalculated into the expected direction of displacements. In the case of bridges, the expected direction is usually vertical. The geometry situation is shown in Figure 1.

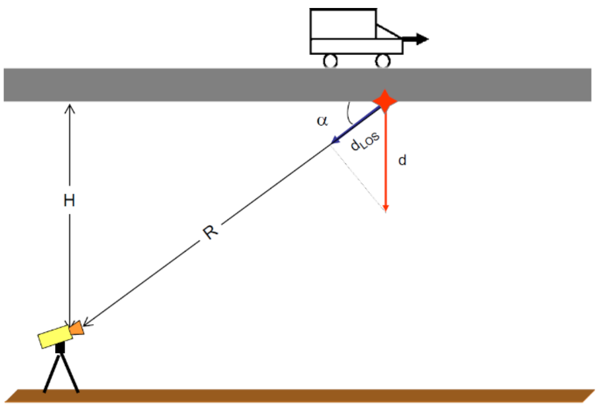


Figure 1. Line of sight movement (d_{LOS}) and expected (calculated) vertical movement (d), R - radar distance from the measured point, and H - radar distance from the measured point in vertical direction.

The assumed (expected) vertical displacement is calculated according to formula:

$$d = d_{LOS}\, R/H \tag{1}$$

However, the assumption of only a vertical displacement may not be fulfilled and is generally not fulfilled. The reason is, for example, that bridges are often nor horizontal nor straight, and then significant longitudinal or transverse deformation occurs at the same time as a result of torsion during vertical deflection and also vehicles generate usually longitudinal and transverse horizontal forces (e.g., braking forces or centrifugal forces) during their passages. In most cases, the longitudinal or transverse horizontal displacements are much smaller than the vertical ones. In some cases, however, horizontal displacements can become significant compared to vertical displacements. Examples of railway bridges with significant values of transversal horizontal displacements are presented in Xiang at al. (2004) and Jin at al. (2016). In Miccinesi at al. (2021) errors from the erroneous assumption of only vertical displacements are pointed out. In other words, these are errors from not taking horizontal displacements into account when determining vertical displacements using the GB-RAR method with only one radar. This error from not taking horizontal displacements into account is discussed in more detail in Olaszek at al. (2021), where it is called an Interpretation Error E_I.

The geometric situation clarifying the origin of the Interpretation Error E_I is shown in Figure 2.

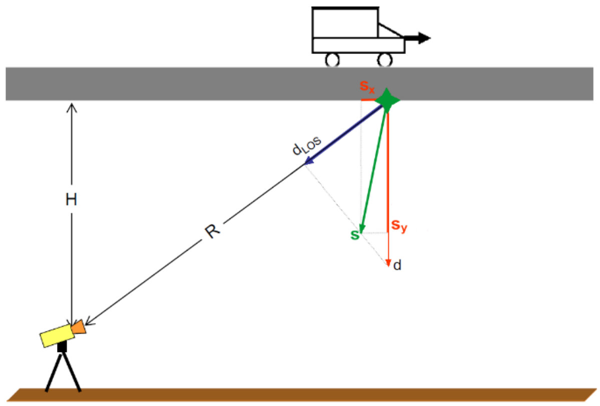


Figure 2. Origin of the Interpretation Error E_I when measuring with only one interferometric radar: s - total displacement; s_y - vertical component of total displacement; s_x - horizontal component of total displacement; d_{LOS} - measured displacement in the range direction; d - calculated vertical displacement; R - radar distance from the measured

point; H - radar distance from the measured point in vertical direction.

In accordance with Olaszek at al. (2021), the Interpretation Error E_I can be expressed as follows:

$$E_I = (d - s_y)/d \quad (2)$$

Then, the Interpretation Error E_I can be calculated based on the geometry shown in Figure 2 according to the relationship:

$$E_I = \frac{s_x}{s_y}\sqrt{\left(\frac{R}{H}\right)^2 - 1} \quad (3)$$

Formula (3) therefore gives the relationship between Interpretation Error E_I and the ratios *R/H* (radar distance from the measured point/radar distance from the measured point in vertical direction) and s_x/s_y (longitudinal or transversal horizontal displacement/vertical displacement). For illustration, with the usual size of the ratio of horizontal displacements to vertical s_x/s_y = 0.10 in practice, the value of Interpretation Error E_I = 23% already at the ratio *R/H* = 2.50. At the ratio *R/H* = 5.00, E_I = 49%. With a greater ratio of horizontal to vertical displacements, which can occur in some cases, the E_I values are even significantly larger. The size of the Interpretation Error can therefore take on very significant values and in common practice can completely invalidate the measurement results and lead to erroneous conclusions regarding the health of the tested structure. The most important finding regarding the influence of the Interpretation Error E_I is that, with some exceptions, it is not possible to rely on the results of measuring vertical displacements with only one radar.

It is therefore necessary to design new procedures for measuring and processing the measured LOS displacements in order to detect and determine the actual directions and magnitudes of the real (total) displacements. The ability to measure by two or more radar systems simultaneously would be able to overcome this shortcoming in probably the most effective way. It is also possible to eliminate this shortcoming with the help of a computational model of the bridge. However, in most cases, it is not available, and even so, its options are limited by uncertain boundary conditions and input parameters.

Simultaneous measurements with two radars are mentioned in the commonly available scientific literature only rarely. One of the first articles dealing with the use of two radars for determining bridge displacements is Dei at al. (2013). The first time the principle of calculation of real (total) displacements when measuring with two radars is given in IDS (2016). The issue of time synchronization of measurements, which is crucial for correct calculation of real displacements, is not mentioned there. From the next literature dealing with the determination of 2D/3D displacements by measuring with two or more radars, ref. Monti-Guarnieri at al. (2018) and Michel and Keller (2021) can be cited. Only in Talich at al. (2023) is a universal solution to the problem of time synchronization presented, and then, above all, a detailed analysis of the accuracy of measurements with two radars, allowing its planning with regard to the required accuracy of the resulting displacements.

The aim of the article is to inform about a suitable strategy for determining dynamic and quasi-static response of bridge structures based on the accuracy analysis of measurement by two radars. The key contribution is the possibility to estimate and plan in advance the achievable accuracy of the resulting displacements for the given radar configurations.

2 Method of GB-RAR with Two Interferometric Radars

Simultaneous measurements with two radars bring up several technical problems that need to be solved. It is mainly a matter of determining the spatial configuration of radars and the measured bridge, which enables calculation of real displacements. Furthermore, time synchronization of both radars causes serious problems as well.

2.1 Data Processing

If we assume that the bridge deck moves along two directions (longitudinally and vertically), it is possible to determine the real displacements and their individual components in vertical plane by simultaneous measurement with two radars. Figure 3 shows two basic configurations of the positions of the two radars when measuring bridges: the radars measure against each other - at the top, or the radars measure from one side of the bridge - at the bottom. Then, Figure 4 shows geometric relations between LOS displacements and real (total) displacements of a point measured from two different radar positions.

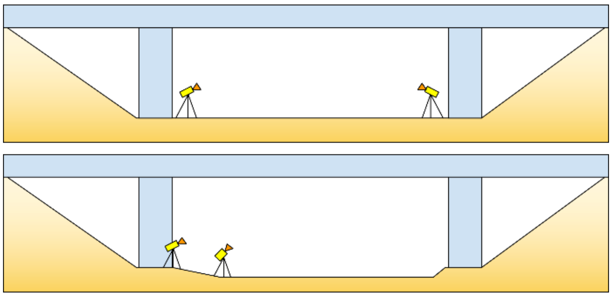


Figure 3. Two basic configurations of the position of the two radars when measuring bridges.

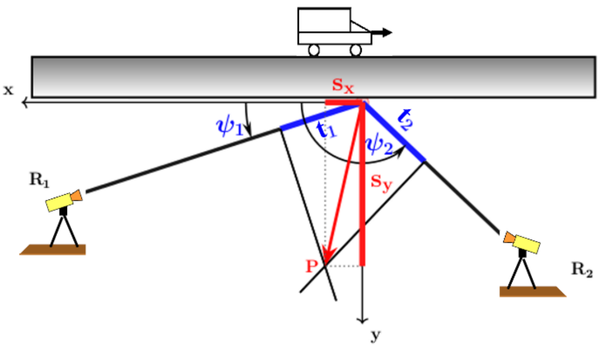


Figure 4. Relationship of displacement vector [s_X,s_Y] to measured LOS displacements t_1, t_2 and vertical angles ψ_1, ψ_2 from radars R_1, R_2 to the monitored point.

The longitudinal and vertical components of the displacement vector are functions of vertical angles of the radar directions and the measured displacements in these directions (LOS). They can be calculated by Formulae (4) (Olaszek at al., 2021), (IDS, 2016) which are derived in Talich at al. (2023).

$$s_X = \frac{t_1 \sin(\psi_2) - t_2 \sin(\psi_1)}{\sin(\psi_2 - \psi_1)}, \quad s_Y = \frac{-t_1 \cos(\psi_2) + t_2 \cos(\psi_1)}{\sin(\psi_2 - \psi_1)} \tag{4}$$

where

t_1, t_2 … measured LOS displacements;

ψ_1, ψ_2 … vertical angles of the radar directions;

s_X, s_Y … components of the displacement vector.

In this way, it is possible to determine the longitudinal and vertical components of the total (real) displacement of the monitored point. However, since the IBIS Data Viewer software supplied with the radar equipment does not allow the evaluation of multi-radar measurements, it is necessary to export the data (LOS displacements) from the IBIS software and, afterwards, process them by some other suitable software.

A serious limitation of this measurement procedure is the interference of the signals of both radars. Interference was observed for any configuration and position of the radars. More interference occurs when the antennas of the radars are facing each other, but it also depends on the object being observed. Its size also depends on the type of radar. Radar IBIS-FS is generally interfered with less than IBIS-S. The size of the interference is also smaller if the value of NumberOfDeadTonesBetweenTwo-Sweeps is as small as possible. This value can be found in the ini file stored with the measurement. The value depends on the Sampling Frequency and the Max distance of the measurement. Interference is expressed by periodic peaks. Their frequency depends on the settings of the radars. If the interference is larger, it needs to be filtered out. If we wanted to measure without interference, the radars would have to communicate directly with each other, which is not possible with ordinary IBIS-S and IBIS-FS radars.

2.2 Time Synchronization of Two or More Radars

When measuring with two radars, there is a practical problem how to recognize displacements measured at the same time in two different time series of the acquired LOS displacements. Therefore, the time series have to be synchronized to find time correspondence of the acquired LOS displacements. If the measurement is performed for example with sampling rate 200 Hz, then the synchronization must be performed with the appropriate accuracy, i.e., ±0.0025 s.

One possible solution of the synchronization is based on identification of maximum deflection values in the two timeseries (Olaszek at al., 2021). Positions of these maxima presumably correspond to the same moment of acquiring them. Hence the synchronization could be performed simply as a time shift obtained after fitting peaks of the two timeseries. There is a serious disadvantage of this method. Deflection values acquired in both time series may not reach their maxima at the same moment. Fitting the time series is therefore only approximate and may not reach the required accuracy ±0.0025 s.

Due to the above disadvantage, more accurate method of synchronization was designed. This method utilizes system times of operating laptops that control measurement processes of the radars and on which the measured data are stored. Synchronizing the two radars therefore means synchronizing the system time of their operating laptops. This method of time synchronization can also be used when synchronizing radar measurements using the GB-RAR method with measurements using other methods, for example using accelerometers or photogrammetry. More practical information about this method of radar time synchronization is given in Talich at al. (2023).

2.3 Accuracy Analysis of Longitudinal and Vertical Component of the Total Displacement

Accuracy analysis of displacements $[s_{X,Y}]$ stems from Equation (4). Covariance matrix of the displacement vector $[s_{X,Y}]$ can be estimated using a well-known formula of error propagation, if precisions of LOS displacements t_1, t_2 and radar directions ψ_1, ψ_2 are given in advance. Therefore, prediction of accuracy of the resulting displacements is possible. The prediction is important namely for planning appropriate placements of the radars in the terrain.

The actual derivation of the extensive resulting formulas for the calculation of accuracy is given only in Talich at al. (2023). Here, due to lack of space, we will only present illustrative examples of the conclusions that flow for practice. Practically can be stated that the accuracy is sensitive to mutual configuration of the radars, the region of interest on the bridge and also on the size and direction of the displacement vector.

An example of mean error ellipses showing the resulting accuracy of total displacements at different locations of the monitored bridge is in Figure 5.

It can be seen that positioning the radars opposite each other gives much more accurate results than positioning them one behind the other, which can only be understood as an emergency solution if the terrain situation does not allow otherwise.

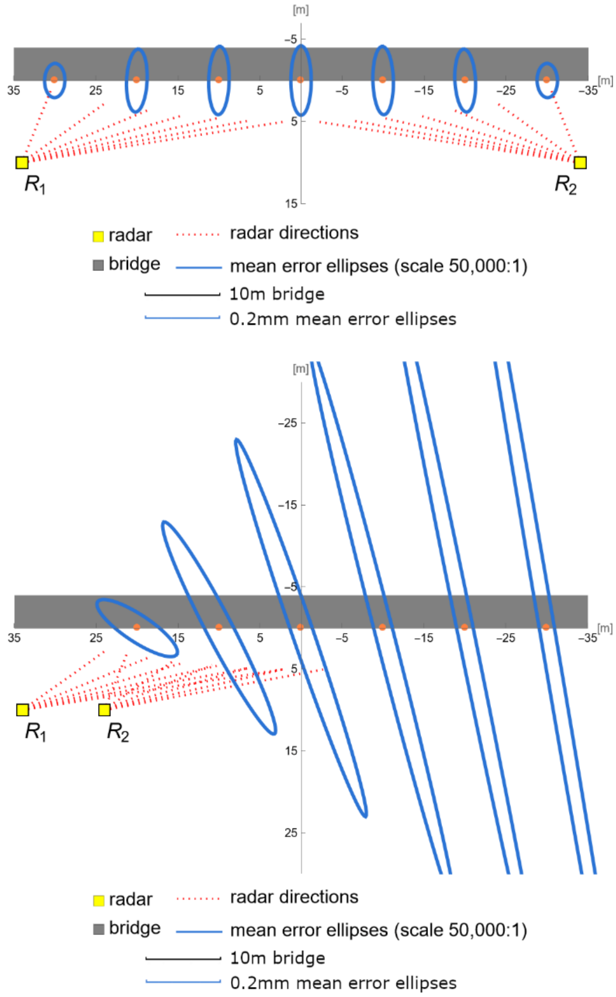


Figure 5. Accuracy of displacement vector $[s_X,s_Y]$=[0.0 mm, 5.0 mm] at various points on the bridge. Positions of the radars are determined with precision 0.2 m. Standard deviation of measured LOS displacements is 0.02 mm. Scale of the mean error ellipses is 50,000 : 1. at the top - radars are placed at opposite ends of the bridge; at the bottom - radars are placed behind each other.

Furthermore, the most accurate results are at the point on the bridge deck where the radar directions are perpendicular to each other. That is, when the radars are positioned opposite each other, the resulting displacements at the edges of the bridge are more accurate than in the middle. This means that high accuracy in determining displacements cannot be achieved for bridges with a small height above the terrain.

An example of mean error ellipses showing the resulting accuracy of the total displacements as a function of their magnitude is shown in Figure 6.

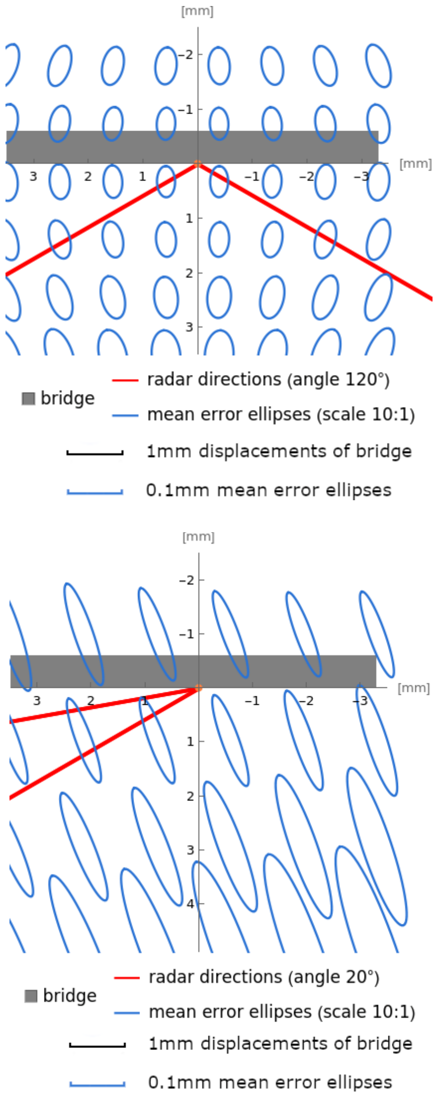


Figure 6. Accuracy of displacement vector $[s_{X,Y}]$ depending on its magnitude. Standard deviation of measured LOS displacements is 0.02 mm. Scale of the mean error ellipses is 10: 1. At the top - radars are placed at opposite ends of the bridge and vertical angles of radar directions are 30° and 150°; at the bottom - radars are placed behind each other and vertical angles of radar directions are 10° and 30°.

Here too, it can be seen that positioning the radars one behind the other gives significantly worse results in terms of achieved accuracy than positioning them opposite each other. At the same time, it can be seen that the accuracy of determining the resulting displacements decreases with an increase in their size. Therefore, for bridges with larger deflection, e.g. longer steel bridges, the accuracy of determining the deflection at the moment of its maximum value during vehicle passage is lower than when it is not passing through. Small deformations due to temperature changes will thus be determined more accurately than large dynamic deformations due to vehicle passage.

3 Experimental Measurement in Order to Verify Theory

In order to verify the above theory, experimental measurements of the new railway steel arch bridge in the Púchov municipality, Slovakia, where the configuration of placing the radars behind each other was used. There were used three different methods:

- highly sensitive piezoelectric accelerometers
- ground-based radar interferometry (GB-RAR)
- photogrammetry method with digital image correlation (DIC)

The experiment was carried out in September 2021 and more details about it are given in Talich at al. (2023).

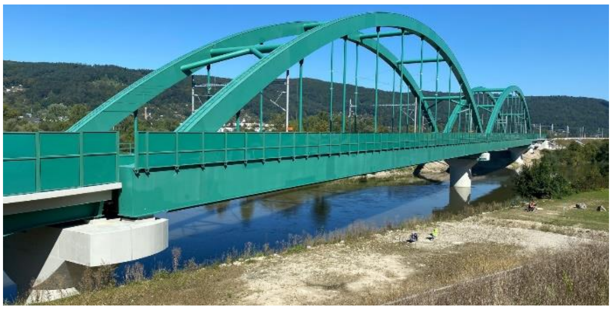


Figure 7. Measured third field with a span of 124.8 m of the railway bridge in Púchov.

Figure 8. Used interferometric radars: Left - Bottom view on the bridge deck with the steel crossbeams from the position of the radar R1; Right - View on both radars: R1 is in the fore-ground, and R2 is in the background in the photo.

On the bridge in Púchov, only one point (crossbeam) in the 1st quarter of the bridge was evaluated from the data measured by both radars for the purposes of this article. This crossbeam corresponds to Rbins R1 23 and R2 41. Comparing radar measurements with accelerometer ones is not appropriate in this case, because used accelerometers Brüel & Kjær type 8344 do not detect the quasi static component of motion that is dominant in the observed bridge dynamic response to the used dynamic load. For this reason, the photogrammetric digital image correlation (DIC) method was chosen to compare the results of the radar measurement with the results of some other experimental method.

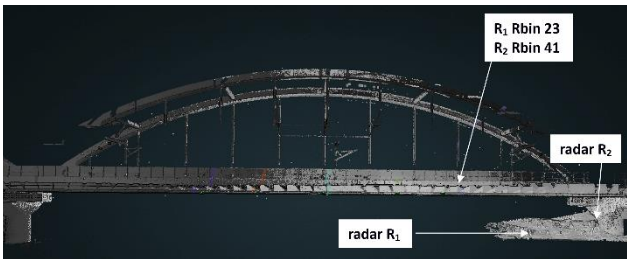
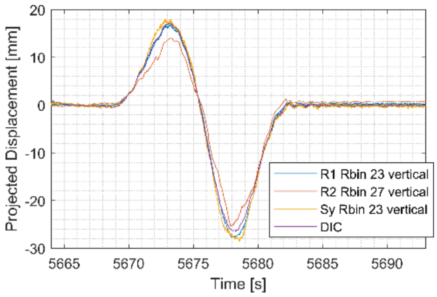
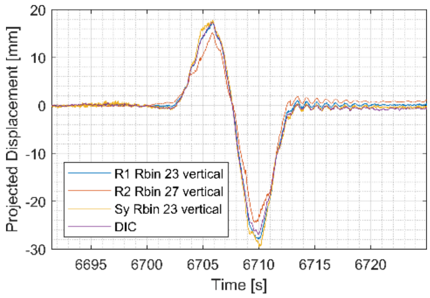


Figure 9. Three-dimensional model of the bridge with color-highlighted Rbins for radars R1 and R2, which were evaluated, and with marked positions of both radars below the structure.

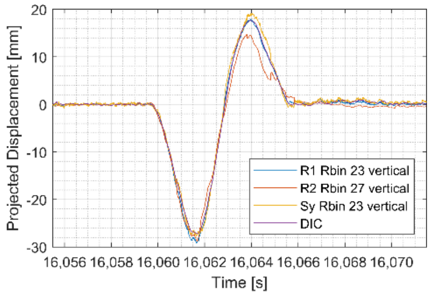
Figure 10 (a)-(c) show comparisons of the separately calculated (1) vertical displacements that were measured by radars R1 and R2 with the vertical (Sy) displacements calculated by combination of both radar measurements of these radars (4) and with results of the DIC measurement at the same location. At the Figure 10 (d)-(f) is the comparisons of vertical displacements biases between radar (Sy) and DIC measurements with the confidence intervals. In this case, the influence of the displacement magnitudes on the magnitude of the confidence intervals is significant.



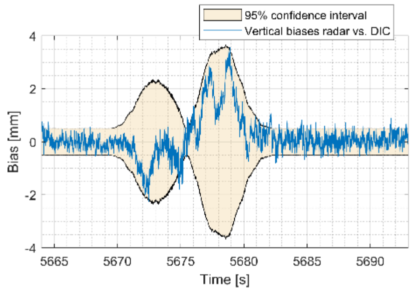
(a)



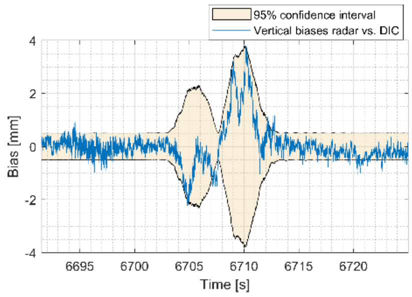
(b)



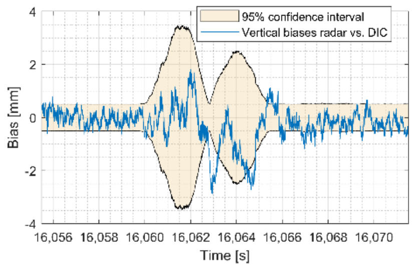
(c)



(d)



(e)



(f)

Figure 10. (a)-(c): comparison of vertical displacements during the passage of the test train including 2 locomotives: (a) at 40 km/h - direction Bratislava; (b) at 50 km/h - direction Bratislava; (c) at 90 km/h - direction Žilina; (d)-(f): comparison of the confidence intervals with biases between radar and DIC measurement of vertical displacements: (d) for the train speed 40 km/h. Percentage of the biases that belong into the 95% confidence interval is 96%; (e) 50 km/h, 94%; (f) 90 km/h, 84%.

Another verification experiment with configuration of placing the radars against each other, which contains more details, is given in Talich at al. (2023).

4 Conclusions

The use of simultaneous measurement by two radars is necessary to eliminate the so-called Interpretation Error E_I, which occurs, with exceptions, always when measuring with only one radar. This error can completely invalidate the achieved results if measured with only one radar. By carrying out an analysis of the accuracy of the results of measurements by two radars, several insights for practice were achieved. The most important of these is that placing radars opposite each other is much more suitable than placing radars behind each other. Further, that it is possible to estimate and plan in advance the achievable accuracy of the resulting displacements for the given specific radar configurations in relation to the bridge structure. The derived formulas for the resulting accuracy of the specified displacements, given only in Talich at al. (2023), can be used in advance to model and calculate the achievable accuracy. This will make it possible to determine the optimal measurement strategy with two interferometric radars and thereby reduce the financial costs of performing measurement and monitoring work.

Acknowledgements

This paper was supported by the project SS07020036 „*Digital automated monitoring of the synergic effects of flooding of closed mines in the Kladno coalfield to manage the risks of the consequences of mining activities and further area development*“, which is co-financed with state support from the Technology Agency of the Czech Republic under the Environment for Life Programme. This project was financed under the National Recovery Plan from the European Recovery and Resilience Facility.

References

Akbar, S.J. (2021). Dynamic monitoring of bridges: Accelerometer Vs microwave radar interferometry (IBIS-S). *J. Phys. Conf. Ser.* 2021, 1882, 012124.

Artese, S. and Nico, G. (2020). TLS and GB-RAR Measurements of Vibration Frequencies and Oscillation Amplitudes of Tall Structures: An Application to Wind Towers. *Applied Sciences*. 2020; 10(7):2237. https://doi.org/10.3390/app10072237

Dei, D., Mecatti, D. and Pieraccini, M. (2013). Static Testing of a Bridge Using an Interferometric Radar: The Case Study of “Ponte Degli Alpini,” Belluno, Italy. *Sci. World J*. 2013, 2013, e504958.

Gentile, C. and Bernardini, G. (2010). An interferometric radar for non-contact measurement of deflections on civil engineering structures: laboratory and full-scale tests. *Structure and Infrastructure Engineering*, 6, 2010 – Issue 5, https://doi.org/10.1080/15732470903068557

IDS Ingegneria Dei Sistemi S.p.A. (2016). *Static and Dynamic testing of bridges: use of IBIS-FS for measuring deformation and identifying modal analysis parameters.* Config.: IBIS FS-PRCS-OUT-DT. N doc.: DT/2016/032. Rev. 1.0. Pisa, 14/04/2016, p 56.

Jin, Z., Pei, S., Li, X. and Qiang, S. (2016). Vehicle-Induced Lateral Vibration of Railway Bridges: An Analytical-Solution Approach. *J. Bridge Eng*. 2016, 21, 04015038.

Liu, X., Tong, X., Ding, K., Zhao, X., Zhu, L. and Zhang, X. (2015). "Measurement of Long-Term Periodic and Dynamic Deflection of the Long-Span Railway Bridge Using Microwave Interferometry," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 8, no. 9, Sept. 2015, doi: 10.1109/JSTARS.2015.2464240.

Luzi, G., Crosetto, M. and Fernández, E. (2017). "Radar Interferometry for Monitoring the Vibration Characteristics of Buildings and Civil Structures: Recent Case Studies in Spain." *Sensors* 2017;17(4):669. https://doi.org/10.3390/s17040669

Miccinesi, L., Beni, A. and Pieraccini, M. (2021). Multi-Monostatic Interferometric Radar for Bridge Monitoring. *Electronics* 2021, 10, 247.

Michel, C. and Keller, S. (2021). Advancing Ground-Based Radar Processing for Bridge Infrastructure Monitoring. *Sensors* 2021, 21, 2172.

Monti-Guarnieri, A., Falcone, P., D’Aria, D. and Giunta, G. (2018). 3D Vibration Estimation from Ground-Based Radar. *Remote Sens*. 2018, 10, 1670.

Olaszek, P., Świercz, A. and Boscagli, F. (2021). The Integration of Two Interferometric Radars for Measuring Dynamic Displacement of Bridges. *Remote Sens*. 2021, 13, 3668.

Pieraccini, M., Fratini, M., Parrini, F. and Atzeni, C. (2006). Dynamic Monitoring of Bridges Using a High-Speed Coherent Radar. *IEEE Transactions on Geoscience and Remote Sensing*, vol. 44, no. 11, Nov. 2006, doi: 10.1109/TGRS.2006.879112. https://ieeexplore.ieee.org/document/1717722

Pieraccini, M., Fratini, M., Parrini, F., Atzeni, C. and Bartoli, G. (2008). Interferometric Radar vs. Accelerometer for Dynamic Monitoring of Large Structures: An Experimental Comparison. *NDT E Int*. 2008, 41, 258–264.

Talich M. (2017). Using Ground Radar Interferometry for Precise Determining of Deformation and Vertical Deflection of Structures. *IOP Conference Series Earth and Environmental Science*, 95(3):032021, 2017, doi: 10.1088/1755-1315/95/3/032021

Talich M. (2018a). Monitoring of horizontal movements of high-rise buildings and tower transmitters by means of ground-based interferometric radar. *Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci.*, XLII-3/W4, 499-504, https://doi.org/10.5194/isprs-archives-XLII-3-W4-499-2018, 2018.

Talich M. (2018b). The Effect of Temperature Changes on Vertical Deflections of Metal Rail Bridge Constructions Determined by the Ground Based Radar Interferometry Method. *IOP Conf. Series: Earth and Environmental Science* 221 (2019) 012076, doi: 10.1088/1755-1315/221/1/012076

Talich, M., Havrlant, J., Soukup, L., Plachý, T., Polák, M., Antoš, F., Ryjáček, P. and Stančík, V. (2023). Accuracy Analysis and Appropriate Strategy for Determining Dynamic and Quasi-Static Bridge Structural Response Using Simultaneous Measurements with Two Real Aperture Ground-Based Radars. *Remote Sens.* 2023, 15, 837. https://doi.org/10.3390/rs15030837

Xiang, J., Zeng, Q. and Lou, P. (2004). Transverse Vibration of Train-Bridge and Train-Track Time Varying System and the Theory of Random Energy Analysis for Train Derailment. *Veh. Syst. Dyn*. 2004, 41, 129–155.

Yu, J., Meng, X., Yan, B., Xu, B., Fan, Q., Xie, Y. (2020). Global Navigation Satellite System-based positioning technology for structural health monitoring: a review. *Struct Control Health Monit*. 2020; 27:e2467. https://doi.org/10.1002/stc.2467